Towards a metalogical appraisal of *fuzzy logic*⁎

Dídac Ramírez Sarrió

Universitat de Barcelona. Facultat de Ciències Econòmiques i Empresarials

Resum: Hi ha un ampli ventall de sistemes coneguts com a *lògica borrosa* que s'utilitzen per basar inferències que inclouen vaguetat. Aquests sistemes operen sobretot en dos camps: el de les aplicacions tecnològiques i el lògic. A conseqüència d'aquesta dualitat l'expressió 'lògica borrosa' admet diversos significats i el seu ús esdevé confús.

El sentit original va ser introduït per L.A. Zadeh (1975) per denotar una nova família de sistemes lògics a fi d'adaptar-se al raonament humà tant com fos possible. Entre les opcions disponibles, la proposada per Zadeh representa l'atac més radical contra la lògica estàndard en considerar que *veritable* i *fals* són ells mateixos predicats borrosos. Malgrat la seva importància, el primer significat de lògica borrosa està en desús entre els practicants dels mètodes borrosos.

El treball té per objecte avaluar la importància i validesa de la lògica creada per Zadeh. Després de situar aquesta lògica dins el context lògic, resumeixo els diversos significats de l'expressió *lògica borrosa*; tot seguit recordo els trets principals de la proposta de Zadeh i discuteixo la seva validesa per a tots els propòsits pràctics; finalment, presento una avaluació metalògica del fons implicat en tal proposta.

Paraules clau: lògica borrosa, vaguetat, coneixement aproximat, raonament aproximat, principi de bivalència, incertesa, coneixement imperfecte.

1. Introduction

Aristotle is commonly considered to have been the first scholar to present an exposition on logic. His treatises on this subject were collectively called *Organon*, a Greek-derived word meaning "tool". Therefore, logic is, in its original and widest meaning, a tool to guarantee the coherence of "reasoning", that is to say, of "an argument in which, certain things being laid down, something other than these necessarily comes about through them" (Topics, I, 1).

⁎ This paper is an updated version of the previous one (2002).

Traditionally, the only *organon* used to discriminate "valid" arguments from those that are not has been classic logic. Nowadays, uncertainty involving subjects such as the future, the infinite, non-sense or vagueness caused alternative logical systems to spread.

With regard to *vagueness,* some years ago I reviewed (1995) the set of logical alternatives. Depending on whether the principle of bivalence (every statement is either true or false) is accepted or rejected, these alternatives can be classified in two main groups. The first one follows the criterion of *precision*: informal arguments of ordinary languages must be purified to make their treatment within standard logic possible. The second one follows the criterion of *adaptation*: standard logic must be modified to such an extent that it can operate with vagueness with no need for this logic to change.

The logical systems that belong to the second group are generically known as *fuzzy logics.* Nevertheless, in spite of this common denomination, two subgroups have to be distinguished, which are called the *super-valued* and the *fuzzy* approaches, respectively. The former, which includes multivalued logics, introduces new truth-values to solve the problems caused in the determination of truth or falsity in doubtful cases. The latter proposes more radical changes in adapting logic to ordinary language, for instance, to deal with approximate reasoning using fuzzy linguistic truth-values instead of the truth-value set of a multivalued-logic.

In this paper, I am concerned with this last approach, the leading exponent of which is Lotfi A. Zadeh. It was he who introduced the original sense of *fuzzy logic* in 1975 to denote a family of new logical systems which aim to adapt to human reasoning as far as possible. Among the several options available to us for approximate reasoning with ordinary languages, Zadeh's represents the most radical attack against standard logic, in considering that *true* and *false* are themselves fuzzy predicates. In spite of its significance, the early sense of fuzzy logic is no longer in use, although not unknown, among practitioners of fuzzy methods.

2. The ambiguity of the term 'fuzzy logic'

There is a wide range of systems, commonly known as *fuzzy logic,* which are used to base inferences that include vagueness on. These systems, which are utilised within the framework of the fuzzy thinking paradigm, operate in two main fields: technological applications and logic. As a result of this duality, the term *fuzzy logic* allows several meanings which give rise to confusing use.

The denomination of *fuzzy logic* has always been ambiguous and can be misleading. As long ago as 1976, B.R. Gaines pointed out three potential meanings: (i) as a basis for reasoning with vague statements, (ii) as a basis for reasoning with vague statements using fuzzy sets theory for the fuzzyfication of logical structures, (iii) as a multivalued logic in which truth-values belong to the interval [0,1] and the *min* and *max* rules of Lukasiewicz's logic are applied.

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This distinction was made before the remarkable success reached by the new fuzzy technologies, whose activities are usually regarded as forming the core of fuzzy logic. Zadeh himself (1994) stated:

> The term fuzzy logic is actually used in two different senses. In a narrow sense, fuzzy logic is a logical system which is an extension of multivalued logic and is intended to serve as a logic of approximate reasoning. But in a wider sense, fuzzy logic is more or less synonymous with the theory of fuzzy sets [...] what is important to recognise is that today the term fuzzy logic is used predominantly in its wider sense.

And S. Haack (1996, p. 232), for her part, has asserted:

> The term 'fuzzy logic' seems to be used, in literature, to refer to two related, but distinct, enterprises: (I) the interpretation of familiar infinitely many-valued logics in terms of fuzzy set theory, and (II) the development, on the basis (I), of a family of new logical systems in which the truth-values are themselves fuzzy sets.

So, at least four meanings of *fuzzy logic* can be distinguished: (α) as a basis for reasoning with vague statements or terms; (β) as technological reasoning with vague terms using the mathematical theory of fuzzy sets; (γ) as a certain kind of multivalued logic; (δ) as a multivalued logic in which truth-values are interpreted in terms of fuzzy set theory; (ϵ) the narrow sense quoted by Zadeh and Haack (II).[1] Here I will reserve the term *fuzzy logic* for (ϵ), henceforth FL.

3. ZADEH'S PROPOSAL. FORMAL COMMENTS

The divergence of FL with regard to classic logic is much more radical than any other logical system. I consider Zadeh's proposal to be the boldest attempt ever made to construct a logic that is able to make inferences of ordinary reasoning within ordinary languages. In (1975), Zadeh summarised what FL consists of:

> Perhaps the simplest way of characterising fuzzy logic is to say that it is a logic of approximate reasoning. As such, it is a logic whose distinguishing features are (i) fuzzy truth-values expressed in linguistic terms, e. g., *true, very true, more or less true, rather true, not true, false, not very true and not very false,* etc.; (ii) imprecise truth tables; and (iii) rules of inference whose validity is approximate rather than

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1. In (1993), B. Kosko included a reference to *fuzzy logic* within the glossary included in the work and distinguished two meanings, more or less corresponding to (β) and (δ) respectively. Some years later, in (1999) Kosko seems to appeal to (α) when he laconically defines *fuzzy logic* as consisting of reasoning by means of fuzzy concepts, although it falls into the category of (β) when he states that fuzzy logic is a branch of Artificial Intelligence. It should be emphasised that, strictly speaking, (α) and (β) do not correspond to the field of logic. Altogether, this gives us an idea of this ambiguity.

exact. In these respects, fuzzy logic differs significantly from standard logical systems ranging from the classical Aristotelian logic to inductive logics and many-valued logic with set-valued truth-values.

To construct an FL, a multivalued logic that verifies *min* and *max* rules for conjunction and disjunction respectively is needed. This gives rise to a family of FL, each one with its own base logic. The logic used by Zadeh is Lukasiewicz's indenumerably many-valued logic (Aleph$_1$), which takes its truth-values from the real unit interval [0,1].

The truth-values set of FL is not [0,1] but a denumerably set of fuzzy sets identified by means of *linguistic labels* like *true*, *very true*, etc. These truth-values are obtained by two kinds of rules, syntactic and semantic, respectively. The syntactic rule allows us to generate the linguistic truth-values set of FL, say **T** = {true, false, not true, very true, very (very true), not very true, not true and not false, true and (not false or not true),...}. The semantic rule is an algorithmic procedure to compute the meaning of the elements of **T**, which are fuzzy (sub)sets of [0,1]. **T** includes one (or more) primary terms, *true* for instance, whose meaning has to be specified beforehand and serves as a basis to determine the meaning of the rest of the elements of **T**.

For example, let **T** be the truth-values set of the basic logic and let μ_τ be a fuzzy membership function μ_τ: **T** $\rightarrow$ [0,1]. The meaning of *true* can be expressed as $\tau = \int \mu_\tau(t)/t$, where the symbol $\int$ denotes the union of the single fuzzy sets {$\mu_\tau(t)/t$} and $\mu_\tau(t)/t$ means that the degree of membership of $t \in$ **T** in the labelled fuzzy set is $\mu_\tau(t)$. So, if $\tau = \{\mu_\tau(\textbf{T})/\textbf{T}\} = \{<0/0>, ..., <0/0.5>, <0.3/0.6>, <0.5/0.7>, <0.7/0.8>, <0.9/0.9>, <1/1>\}$,[2] it can be seen that *true* = 0.3/0.6 + 0.5/0.7 + 0.7/0.8 + 0.9/0.9 + 1/1. The meaning of the remaining linguistic values of **T** is obtained from the prefixed one; thus, *very true* = (*true*)^2 = 0.09/0.6 + 0.25/0.7 + 0.49/0.8 + 0.81/0.9 + 1/1.

Once **T** has been determined, the next step is to operate with its elements, in other words extend the definitions of the connectives of the base logic to FL. This is achieved by applying the extension principle for fuzzy sets. The result obtained is a fuzzy truth-value which, in most cases, will have to be submitted to linguistic approximation, which is not unique, to fit it to the **T**-values.

Moreover, the meaning of the statements, connectives and truth-values is variable, not fixed, or, what amounts to the same thing, the meaning is of local validity, rather than universal. This is the reason why FL can be viewed as a *local logic*. Hence, the inference process has a semantic character rather than a syntactic one: in FL, the conclusion depends on the meaning assigned to the fuzzy sets that appear in the set of premises, e.g., the conclusion derived from the two premises P_1: 'John is young' and P_2: 'John and Peter are of roughly the same age' depends on the meaning of 'young' and 'of roughly the same age', both expressions being represented by means of fuzzy sets of R and R², respectively. Let us suppose that we conclude P_3: 'Peter is

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2. For the sake of simplicity, we take **T** as an hendeca-valued set instead of the unit interval [0,1].

more or less young'. We could ask for the "truth" of both premises and the conclusion and to estimate that P_1 is very true, P_2 is rather true and then, applying the methods of *approximate reasoning*, we could conclude that P_3 is true. Therefore, the inference is imprecise, which is the essential characteristic of ordinary reasoning to which FL tries to adapt.

Consequently, FL is the result of a double weakening of the prior assumptions of standard classic logic. Firstly, because of the rejection of the principle of bivalence and the law of excluded middle, which gives rise to a multivalued logic and, closely related to it, to a membership function that allows us to interpret the predicates. Secondly, because of the variability of the meaning assigned both to the truth-values and the connectives, which makes the logical inference itself imprecise.

4. A COMPARATIVE ASSESSMENT OF FUZZY LOGIC

As we saw in the previous section, dealing with *linguistic labels* is the same as doing so with fuzzy sets of the unit interval $[0,1] \in R$, while Lukasiewicz's (Aleph$_1$) logic deals with the real number of $[0,1]$. Apparently, the new method offers an erroneous way because of its greater complexity. Zadeh (1975) gives two reasons to justify this preference for the use of linguistic truth-values instead of numerical ones.

> First, the truth-value set of Aleph$_1$ is a continuum, whereas that of fuzzy logic (FL) is a countable set. More importantly, in most applications to approximate reasoning, a small finite subset of the truth-values of FL would, in general, be sufficient because each truth-value of FL represents a fuzzy subset rather than a single element of $[0,1]$. Thus, we gain by trading the large number of simple truth-values of Aleph$_1$ for the small number of less simple truth-values of FL.

Hitherto, this justification has only alluded to the advantages encountered when operating with both a numerable set and a few, albeit complex, truth-values, as opposed to operating with an indenumerable set including simpler values. Thus, the argument is not conclusive, since it could be answered that one might have a set with a few truth-values without leaving the framework of a basic multivalued logic. The second reason adduced by Zadeh is more substantial:

> [...] approximate reasoning deals, for the most part, with propositions which are fuzzy rather than precise, e.g., "Vera is *highly intelligent*", "Douglas is *very inventive*", "Berkeley *is close to San Francisco*", "It is *very likely* that Jean Paul will succeed", etc. Clearly, the fuzzy truth-values of FL are more commensurate with the fuzziness of such propositions than the numerical truth-values of multivalued logic.

Nevertheless, it is not clear that resorting to commensurability gives us any weighty justification for linguistic truth-values. Why should we consider a fuzzy valuation as 'rather true' to be more adequate than a numerical one of '0.7'?

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Actually we could argue quite the opposite in accordance with the following two remarks. Firstly, precisely because of vagueness we should be careful not to produce new cases of lack of precision; secondly, within a hendeca-valued logic we can associate the numerical value '0.7' to the 'rather true' linguistic fuzzy value without introducing so much complexity.

Therefore, the reasons adduced by Zadeh are not strong enough to adopt FL. If we add that in FL inferences are themselves imprecise, and hence there is no place for the truth-functionality which holds in multivalued logic, it is not surprising that FL has not been taken into account even among the staunchest upholders of the fuzzy paradigm such as B. Kosko. As an alternative both to standard and multivalued logic, Zadeh's attempt is, to all intents and purposes, a failed attempt.

5. METALOGICAL COMMENTS ON FUZZY LOGIC

In spite of the previous conclusion, we have to pay special attention to the background implied in Zadeh's proposal. This background is related to the concern, shared by logicians and linguisticians alike, to offer a semantic system which is suitable for all concepts of ordinary languages.

This concern is closely connected with two metalogical questions, relating to the conception of truth and the nature of logic, respectively. In other words, Zadeh offers us a radical answer to F.C.S. Schiller's claim (1912, p. 8):

> Formal logic is [...] incapacitated by its self-imposed limitation from dealing with the problems of actual thinking and from rationally interpreting the conception of truth implied in such thinking... We need, in short, a second Logic which will be applicable to life and relevant to actual thought.

J. Balmes would thus seem to agree with such a claim (1845, c.22, §60):

> Hay verdades de muchas clases porque hay realidad de muchas clases; hay también muchos modos de conocer la verdad. No todas las cosas se han de mirar de la misma manera, sino del modo que cada una de ellas se ve mejor. Al hombre le han sido dadas muchas facultades. Ninguna es inútil. Ninguna es intrínsecamente mala. La esterilidad o malicia les vienen de nosotros, que las empleamos mal. Una buena lógica debiera comprender al hombre entero; porque la verdad está en relación con todas las facultades del hombre. Cuidar de la una y no de la otra es a veces esterilizar la segunda y malograr la primera.

In (2004) I conjectured that J. Balmes, who has been placed in the same school of thought as the pragmatism of Charles S. Peirce and William James, free of the natural obstacles imposed by the social and intellectual context of his times, would never resigned himself to being swayed by an extremely rigid logic. No matter how we look at it, it is really difficult to accept a logic that allows the validity of the paradoxical

material implication. Moreover, if statements are compelled by the principle of bivalence to be either true or false, or they are at least allowed, as an exception to the rule, to have no truth-value because they are nonsense or for any other reason, what can we say about statements with vague terms, e.g. 'John is tall'? To determine its truth or falsity we should state the exact limit from which we can decide if a person is or not tall; but this limit, on the one hand, will depend on the context, since it will be different for the Batusi and for Pygmies, and, on the other hand, it involves the measurement problems of borderline cases.

Therefore, a numerical value belonging to [0,1] will be assigned to the statement 'John is tall' according to its degree of membership to the fuzzy set made up for the extension of the predicate "tall", i.e. of the "more or less" tall individuals. This corresponds to the first level of FL.

The technical term *hedge* is used to refer to the elements of this fuzzy set. This term was introduced into semantics by G. Lakoff (1972) to denote a modifier of vague predicates. A few examples of "hedges" are: *very, more or less, mutatis mutandis, essentially, strictly speaking,* etc. Lakoff proposed that fuzzy sets theory should be applied to define them by considering that the generation of vague statements through mathematical calculations is thus allowed. Thus, we could consider that the statement 'John is rather tall' is "true" to a degree of 0'9 because <0'9/John> belongs to the fuzzy set of the rather tall persons, while 'John is very (very tall)' would be considered "true" to a degree of 0'2 if <0'2/John> belongs to the set of the persons that are very tall.

Therefore, what Zadeh proposes is to include *true* in the same category as *tall* so that it can also be modified by semantic hedges that label the corresponding fuzzy sets. The question is whether this inclusion is pertinent or not.

This question can be posed in the following way: is truth a quantitative character, to wit, are its modalities capable of increasing and decreasing the same as height, colour, weight, intelligence, goodness, etc.? It is the difficulty to express the modalities of some quantitative characters such as colour, flavour, etc. with precision that lies at the origin of modalities that admit degrees such as tall, white, red, sweet, bitter, etc., which give rise to *secondary characters* such as height, whiteness, redness, sweetness, bitterness, etc. The modalities of these secondary characters are determined by hedges. Is truth one of them, namely, merely the abstraction of *true*? Does truth allow degrees?

S. Haack (1996) opines that the behaviour of the modifiers 'rather' and 'very' with 'true', far from supporting the hypothesis that true is a predicate of degree, suggests that it is an absolute predicate. In other writings I have also stated that truth is a categorical predicate which does not admit modification. In that sense I have argued (1995, 1997) that '... is true' does not correspond to the paradigm of '... is tall', but to the '... is identical', or '... has hit the mark', so, strictly speaking, just as an object cannot be more or less identical, neither do we hit the mark more or less, and neither is a statement more or less true. From this, I have concluded that "rather tall" does

not exclude "tall", "rather true" does exclude "true": at most, a statement labelled as "rather true" can be near the truth and be accepted as founding approximate knowledge, but in actual fact such a statement remains absolutely false.

Now, however, I am not so categorical with regard to this question since I am fully aware of the complexity of the issues involved, such as the problem of truth, the nature of reality and the significance of vagueness in the real world.

In any case, regardless of whether *de jure* truth and falsity are a matter of degree or not, *de facto* there is no doubt that within the scope of human reasoning they are generally and practically accepted to be vague concepts. Therefore, the pertinent questions are: is a Logic such as the one called for by Schiller, a Logic which will be applicable to life and relevant to actual thought, needed?, and does a Logic that does not depend on technology have any use in today's world?

6. CONCLUSION

The term fuzzy logic is ambiguous. Today, it generically denotes a form of reasoning by means of fuzzy concepts, but in a narrow meaning it represents an unsuccessful attempt to offer a logic suitable for approximate reasoning with ordinary languages.

Because of the different significance that researchers have attached to fuzzy logic, its meaning has been misunderstood and it has been confused with both multivalued logic and technological reasoning in a context of vague knowledge. In this paper I have pointed out this fact and have briefly explained the main features of the fuzzy logic proposed by L.A. Zadeh. After arguing about the availability of such a logic as a tool for approximated reasoning, two questions remain open: whether there are degrees of truth or not, and whether a logic applicable to life is needed. In any case, Zadeh's fuzzy logic has to be regarded as a renewed, albeit failed, attempt of a difficult joint venture which, in my view, deserves to be successful in order to answer both Schiller's and Balmes' claims.

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